UK Junior Mathematical Olympiad 2013

Organised by The United Kingdom Mathematics Trust

Tuesday 11th June 2013

RULES AND GUIDELINES :
READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

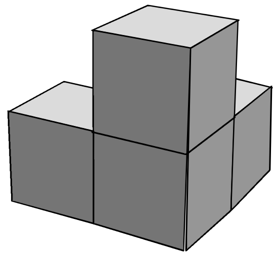
Section A

A1 What is the value of $\sqrt{3102 - 2013}$?

A2 For how many three-digit positive integers does the product of the digits equal 20?

A3 The solid shown is made by gluing together four 1 cm × 1 cm × 1 cm cubes.

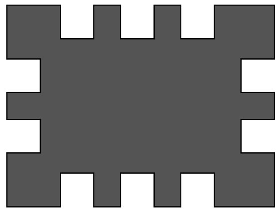
What is the total surface area of the solid?



A4 What percentage of $\frac{1}{4}$ is $\frac{1}{5}$?

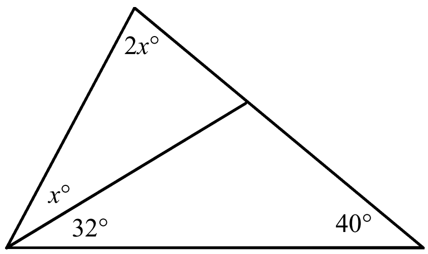
A5 Sue has a rectangular sheet of paper measuring 40 cm × 30 cm. She cuts out ten squares each measuring 5 cm × 5 cm, as shown. In each case, exactly one side of the square lies along a side of the rectangle and none of the cut-out squares overlap.

What is the perimeter of the resulting shape?



A6 I want to write a list of integers containing two square numbers, two prime numbers, and two cube numbers. What is the smallest number of integers that could be in my list?

A7 Calculate the value of x in the diagram shown.



A8 The area of a square is 0.25 m^2. What is the perimeter of the square, in metres?

A9 Each interior angle of a quadrilateral, apart from the smallest, is twice the next smaller one. What is the size of the smallest interior angle?

A10 A cube is made by gluing together a number of unit cubes face-to-face.
The number of unit cubes that are glued to exactly four other unit cubes is 96.

How many unit cubes are glued to exactly five other unit cubes?

Section B

Your solutions to Section B will have a major effect on your JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1 How many numbers less than 2013 are both:

(i) the sum of two consecutive positive integers; **and**

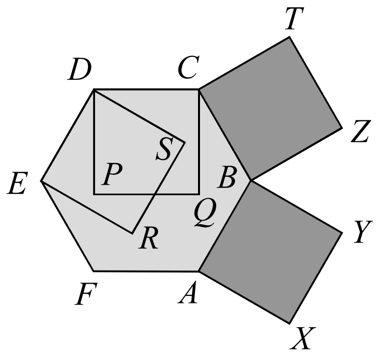
(ii) the sum of five consecutive positive integers?

B2 Pippa thinks of a number. She adds 1 to it to get a second number. She then adds 2 to the second number to get a third number, adds 3 to the third to get a fourth, and finally adds 4 to the fourth to get a fifth number.

Pippa's brother Ben also thinks of a number but he subtracts 1 to get a second. He then subtracts 2 from the second to get a third, and so on until he too has five numbers.

They discover that the sum of Pippa's five numbers is the same as the sum of Ben's five numbers. What is the difference between the two numbers of which they first thought ?

B3 Two squares $BAXY$ and $CBZT$ are drawn on the outside of a regular hexagon $ABCDEF$, and two squares $CDPQ$ and $DERS$ are drawn on the inside, as shown.



Prove that $PS = YZ$.

B4 A regular polygon P with n sides is divided into two pieces by a single straight cut. One piece is a triangle T, the other is a polygon Q with m sides.

How are m and n related?

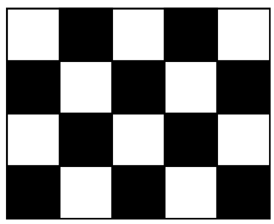
B5 Consider three-digit integers N with the two properties:

(a) N is not exactly divisible by 2, 3 or 5;

(b) no digit of N is exactly divisible by 2, 3 or 5.

How many such integers N are there?

B6 On the 4×5 grid shown, I am only allowed to move from one square to a neighbouring square by crossing an edge. So the squares I visit alternate between black and white. I have to start on a black square and visit each black square exactly once. What is the smallest number of white squares that I have to visit? Prove that your answer is indeed the smallest.



(If I visit a white square more than once, I only count it as one white square visited).